

Abel-Jacob: map.

X smooth proj/ \mathbb{C} $J^r \cong H^{2r-1}(X, \mathbb{C}) / F^r H^{2r-1} + H^{2r-1}(X, \mathbb{Z})$

$A^r =$ codim r -cycle alg. equiv. to 0.

$$\Theta: A^r(X) \rightarrow J^r(X)$$

Thm: Let X be a smooth quartic 3-fold
Then $\Theta: A^2(X) \rightarrow J(X)$ is an isomorphism.

We say Θ is an isogeny if Θ is surjective w/
finite kernel.

Lemma: $\pi: Y \rightarrow X$ blow-up along a sm.
subvariety. $\dim X = \dim Y = 3$.

Θ_X is an isogeny $\Leftrightarrow \Theta_Y$ is an isogeny.

pf:

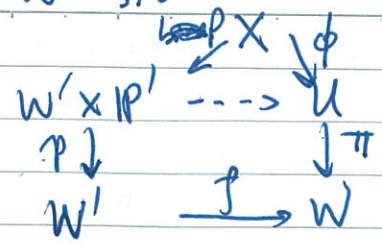
Lemma: $\pi: Y \rightarrow X$ gen. finite deg d .

Θ_Y isogeny $\Rightarrow \Theta_X$ is an isogeny.

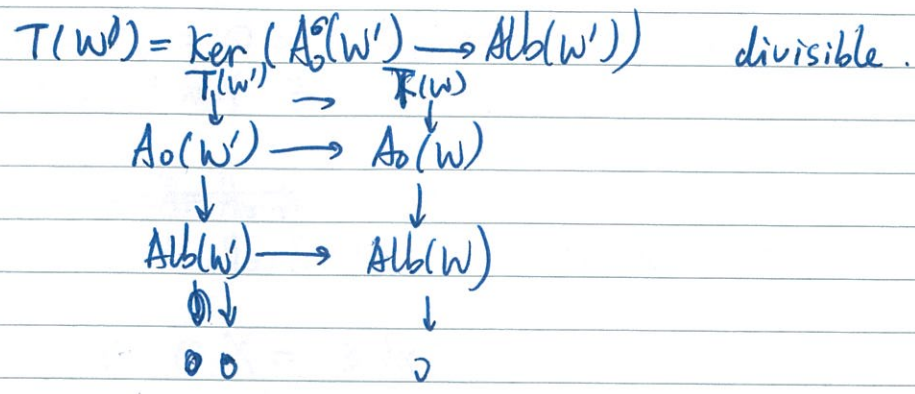
pf:

$$\begin{array}{ccccccc}
 0 \rightarrow \text{Ker } \Theta_Y & \rightarrow & A^2(X) & \rightarrow & J(X) & \rightarrow & 0 \\
 \downarrow & & \downarrow & & \downarrow \pi^* & & \\
 0 \rightarrow \text{Ker } \Theta_X & \rightarrow & A^2(Y) & \rightarrow & J(Y) & \rightarrow & 0 \\
 \downarrow & & \downarrow & & \downarrow \pi_* & & \\
 0 \rightarrow \text{Ker } \Theta_X & \rightarrow & A^2(X) & \rightarrow & J(X) & \rightarrow & 0
 \end{array}$$

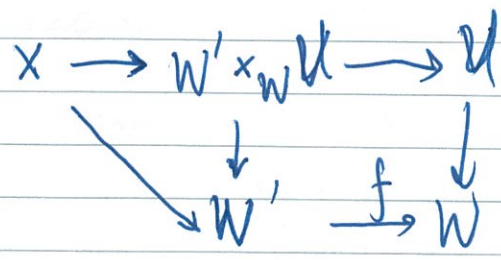
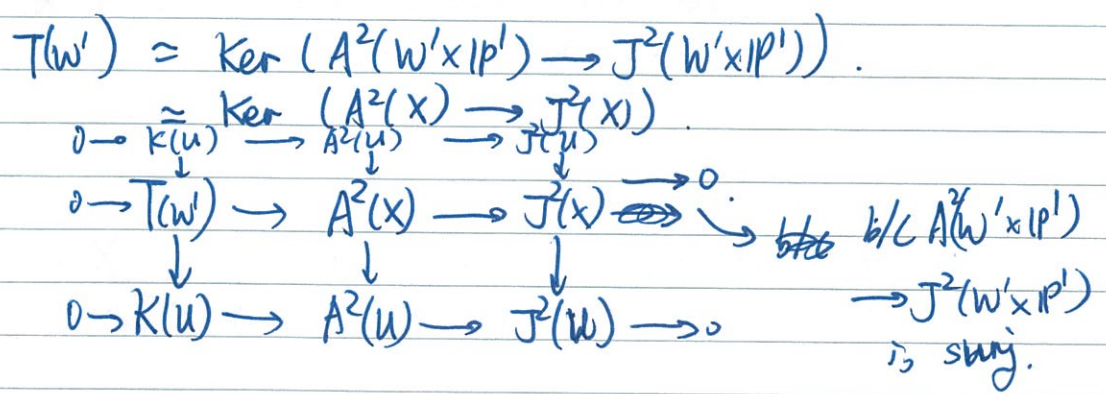
pf. $\exists W'$ s.t



Remk: $A_0(W) \rightarrow Alb(W)$
has huge kernel!



$f_* T(W') \subset T(W)$ is finite but also divisible!
 $\Rightarrow f_* T(W') = 0'$



Geometric construction.

$$x = [1, 0, 0, 0, 0] \in \mathbb{P}^4 \quad x \in X.$$

$$x_0^3 L(x_1, \dots, x_4) + x_0^2 Q(x_1, \dots, x_4) + x_0 C(\dots) + \dots = 0.$$

Assume $L = Q = 0$ defines a sm conic in \mathbb{P}^2
(true if $x \in X$ is general).

~~$y \in \mathbb{P}^3(y_1, y_2, y_3, y_4)$ s.t.~~

$$y = [y_1, y_2, y_3, y_4] \text{ s.t. } L(y) = Q(y) = 0.$$

$$\text{Line } \overline{xy} \cap X = 3 \cdot \{x\} + \{y\}.$$

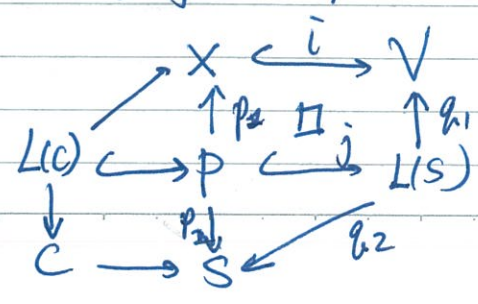
\Rightarrow conic bundle over $X \rightarrow X$.
 $(x, y) \mapsto y$.

\exists a ruled surface $\Sigma \subset X$ s.t. $\Sigma \rightarrow X$ is dominant.
 $\Sigma \rightarrow X$

$V \subset X$ a general quartic 4-fold in \mathbb{P}^5
 $X \cap V \cap H \subset V \subset \mathbb{P}^5$.

$S =$ Fano variety of lines of V .

S is a sm ^{3-fold} surface b/c V is general.



$P \rightarrow S$ is birational. (If C is smooth. $P = \text{Bl}_C S$.)

(*) $\pi_{1*}: H^3(P, \mathbb{Z}) \rightarrow H^3(X, \mathbb{Z})$ is surj.

If P is singular, apply to a resolution or $H_3(P, \mathbb{Z})$?

(**) $\pi_{1*} \pi_2^* H^3(S, \mathbb{Z}) = 0$ in $H^3(X, \mathbb{Z})$.

$$\begin{aligned} \pi_{1*} \pi_2^* H^3(S, \mathbb{Z}) &= \pi_{1*} \left(H^3(S, \mathbb{Z}) \xrightarrow{j^*} H^3(L(S), \mathbb{Z}) \xrightarrow{j^*} H^3(P, \mathbb{Z}) \right) \\ &= \pi_{1*} j^* q^* H^3(S, \mathbb{Z}) \\ &= i^* q_{1*} q_2^* H^3(S, \mathbb{Z}) = 0. \end{aligned}$$

Lemma: Let V be a smooth proj. variety / \mathbb{C} .
 $\dim V = d+1$.

$\phi: W \rightarrow V$ proper morphism. gen. finite.
 W int.

$X \hookrightarrow V$ sm. hyperplane section.

$Y = \phi^{-1}(X)$. then

$$\text{Im } \phi_*: H_d(Y, \mathbb{Z}) \rightarrow H_d(X, \mathbb{Z})$$

contains $\text{Ker } H_d(X, \mathbb{Z}) \rightarrow H_d(V, \mathbb{Z})$.